Quantum field theory of the van der Waals friction

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Abstract

The van der Waals friction between two semi-infinite solids, a small neutral particle and semi-infinite solid is reconsidered on the basis of thermal quantum field theory in Matsubara formulation. The friction calculated in this approach is in agreement with the friction calculated in the framework of dynamical modification of the Lifshitz theory with use of the fluctuation-dissipation theorem. This solves the problem about the applicability of the Lifshitz theory to the dynamical situation. In quantum field theory the calculation of the friction to linear order in the sliding velocity is reduced to the finding of the equilibrium Green functions. Thus this approach can be extended for bodies with complex geometry. We show that the van der Waals friction can be measured in non-contact friction experiment using state-of-the-art equipment.

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1 Introduction

A great deal of attention has been devoted to the problem of non-contact friction between nanostructures, including, for example, the frictional drag force between electrons in two-dimensional quantum wells [1, 2, 3], and the friction force between an atomic force microscope tip and a substrate [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. This is because of it importance for ultra-sensitive force detection experiments. The ability to detect small forces is inextricably linked to friction via the fluctuation-dissipation theorem. According to this theorem, the random force that makes a small particle jitter would also cause friction if the particle was dragged through the medium. For example, the detection of single spins by magnetic resonance force microscopy [14] , which has been proposed for three-dimensional atomic imaging [15] and quantum computation [16], will require force fluctuations (and consequently the friction) to be reduced to unprecedented levels. In addition, the search for quantum gravitation effects at short length scale [17], and future measurements of the Casimir and van der Waals forces [18], may eventually be limited by non-contact friction effects.

In non-contact friction the bodies are separated by a potential barrier thick enough to prevent electrons or other particles with a finite rest mass from tunneling across it, but allowing interaction via the long-range electromagnetic field, which is always present in the gap between bodies. The presence of an inhomogeneous tip-sample electric fields is difficult to avoid, even under the best experimental conditions [6]. For example, even if both the tip and the sample were metallic single crystals, the tip would still have corners, and more than one crystallographic plane exposed. The presence of atomic steps, adsorbates, and other defects will also contribute to the spatial variation of the surface potential. This is referred to as “patch effect”. The surface potential can also be easily changed by applying a voltage between the tip and the sample. An inhomogeneous electric field can also be created by charged defects embedded in a dielectric sample. The relative motion of the charged bodies will produce friction which will be denoted as the *electrostatic friction*.

The electromagnetic field can also be created by the fluctuating current density due to thermal and quantum fluctuations inside the bodies. This fluctuating electromagnetic field gives rise to the well-known long-range attractive van der Waals interaction between two bodies [19, 20], and is responsible for radiative heat transfer. If the bodies are in relative motion, the

same fluctuating electromagnetic field will give rise to a friction which will be denoted as the *van der Waals friction*.

The origin of the van der Waals friction is closely connected with the van der Waals interaction. The van der Waals interaction arises when an atom or molecule spontaneously develops an electric dipole moment due to quantum fluctuations. The short-lived atomic polarity can induce a dipole moment in a neighboring atom or molecule some distance away. The same is true for extended media, where thermal and quantum fluctuation of the current density in one body induces a current density in other body; the interaction between these current densities is the origin of the van der Waals interaction. When two bodies are in relative motion, the induced current will lag slightly behind the fluctuating current inducing it, and this is the origin of the van der Waals friction.

The van der Waals interaction is mostly determined by exchange by virtual photons between the bodies (connected with quantum fluctuations), and does not vanish even at zero temperature. On the other hand, the van der Waals friction, at least to lowest order of perturbation theory, and to linear order in the sliding velocity, is determined by exchange of real photons, and vanishes at zero temperature.

To clarify the origin of the van der Waals friction let us consider two flat parallel surfaces, separated by a sufficiently wide vacuum gap, which prevents electrons from tunneling across it. If the surfaces are in relative motion (velocity v) a frictional stress will act between them. This frictional stress is related with an asymmetry of the reflection coefficient along the direction of motion; see Fig.1. If one body emits radiation, then in the rest reference frame of the second body these waves are Doppler shifted which will result in different reflection coefficients. The same is true for radiation emitted by the second body. The exchange of "Doppler shifted photons" is the origin of van der Waals friction.

From the point of view of the quantum mechanics the van der Waals friction originates from two types of processes:(a) Photons are created in each body with opposite momentum and the frequencies of these photons are connected by $vq_x = \omega_1 + \omega_2$, where q_x is the momentum transfer. (b) A photon is annihilated in one body and created in another. The first process (a) is possible even at zero temperature, and it gives rise to a friction force which depends cubically on sliding velocity [21, 22]. The second process (b) is possible only at finite temperatures, and gives rise to a friction which depends linearly on the sliding velocity. Thus, process (b) will give the main

contribution to the friction at sufficiently high temperatures, and at not too large velocities.

In contrast to the van der Waals interaction, for which theory is well established, the field of van der Waals friction is still controversial. As an example, different authors have studied the van der Waals friction between two flat surfaces in parallel relative motion using different methods, and obtained results which are in sharp contradiction to each other. The first calculation of van der Waals friction was done by Teodorovich [23]. Teodorovich assumed that the force of friction can be calculated as the ordinary van der Waals force between bodies at rest, whose dielectric function depend on the velocity due to the Doppler shift. However, from the dynamical modification of the Lifshitz's theory it follows [22] that it is not true. Later the same approach was used by Mahanty [24] to calculate the friction between molecules. Both theories predict wrong nonzero friction at absolute zero of temperature to linear order in the sliding velocity. The same nonzero linear friction at zero temperature was predicted in [25, 26]. In [27] it was shown that the basic equation in [25, 26] is incorrect, and a correct treatment gives a vanishing linear friction at $T = 0$K. Schaich and Harris developed a theory [28] which describes the dependence of friction on the temperature T and on the separation d. However in their calculations they made the series of unphysical approximations, and for the Drude model their final formula for the friction for parallel relative motion gives a divergent result. The friction obtained in [29, 30, 31] vanishes in the limit of infinite light velocity $c \to \infty$. However, at least for short distances, one can neglect by retardation effects when calculating the van der Waals friction, as well as van der Waals interaction. Pendry [21] assumed zero temperature and neglected retardation effects, in which case the friction depends cubically on the velocity. Persson and Zhang [32] obtained the formula for friction in the limit of small velocities and finite temperature using a simple quantum mechanical approach, again neglecting retardation effects. In Ref.[27] Volokitin and Persson developed a theory of the van der Waals friction based on the dynamical modification of the well known Lifshitz theory [19] of van der Waals interaction. In the nonretarded limit and for zero temperature this theory agrees with the results of Pendry [21]. Similarly, in the nonretarded limit and for small sliding velocity this theory agrees with the study of Persson and Zhang [32]. In [9, 10] the theory was extended to two flat surfaces in normal relative motion. For the case of resonant photon tunneling between surface localized states, normal motion results in drastic difference from parallel relative motion. It was shown that

the friction may increase by many orders of magnitude when the surfaces are covered by adsorbates, or can support low-frequency surface plasmons. In this case the friction is determined by resonant photon tunneling between adsorbate vibrational modes, or surface plasmon modes. When one of the bodies is sufficiently rarefied, this theory gives the friction between a flat surface and a small particle, which in the nonretarded limit agrees with the results of Tomassone and Widom [33]. A theory of the van der Waals friction between a small particle and flat surface, which takes into account screening, nonlocal optic effects, and retardation effects, was developed in [27].

Dorofeyev *et.al.* [4] claim that the non-contact friction observed in [4, 5] is due to Ohmic losses mediated by the fluctuating electromagnetic field. This claim is controversial, however, since the van der Waals friction for good conductors like copper has been shown [22, 27, 34] to be many orders of magnitude smaller than the friction observed by Dorofeyev *et.al.*. In [35] it was proposed that the non-contact friction observed in Ref.[6] can be explained by the van der Waals friction between the high resistivity mica substrate and silica tip. However in the experiment the mica substrate and silica tip were coated by gold films thick enough to completely screen the electrodynamic interaction between the underlying dielectrics.

At small separation $d \sim 1$nm, resonant photon tunneling between adsorbate vibrational modes on the tip and the sample may increase the van der Waals friction by seven orders of magnitude in comparison with the good conductors with clean surfaces [9, 10]. However, the distance dependence ($\sim 1/d^6$) is stronger than observed experimentally [6].

Recently, a theory of non-contact friction was suggested where the friction arises from Ohmic losses associated with the electromagnetic field created by moving charges induced on the surface of the atomic force microscope tip by the bias voltage or spatial variation of the surface potential [11, 12]. It was shown that the electrostatic friction can be greatly enhanced if there is an incommensurate adsorbate layer which can exhibit acoustic vibrations. This theory gives a tentative explanation for the experimental non-contact friction data [6].

Although at present there are many theories of the van der Waals friction, which are frequently contradict to each other, the rigorous theory based on the quantum field theory is still not developed. For the van der Waals interaction such quantum field theory was developed in Ref.[20]. On the base of this theory were solved some problems which can not be solved in the Lifshitz theory of the van der Waals interaction. Due to of great importance

of the van der Waals friction for the understanding of the origin of the non-contact friction there is strong demand for the creation of the rigorous theory of the van der Waals friction.

This article is organized as follows. In Section 2 we present a short overview of the basic idea of the quantum field theory of the van der Waals friction. We apply this theory for the calculation of the van der Waals friction between two semi-infinite solids (Sec.3), and a small particle and semi-infinite solid (Sec.4), for parallel and normal relative motion. These calculations confirm early results obtained with use of the dynamical modification of the Lifshitz theory and the fluctuation-dissipation theorem. Thus these calculations solves the problem of applicability of the Lifshitz theory for bodies in relative motion. The quantum field theory of the van der Waals friction is more general and can be applied for bodies with complex geometry. In Sec.5 we show that the van der Waals friction can be greatly enhanced for high-resistivity metals, dielectrics with strong absorption in low-frequency region, and for two- dimensional systems, e.g. 2D-electron systems on the dielectric substrate, or incommensurate layers of adsorbed ions exhibiting acoustic vibrations. The origin of this enhancement is related to the fact that screening in 2D-systems is much less effective than for 3D-systems. The fluctuating charges will induce "image" charges in the 2D-system. Because of the finite response time during relative motion of the bodies this "image" charge will lag behind the charge which induce it, and this results in a force friction acting on the bodies. The weaker screening in 2D-systems will results in larger lag of the "image" charge in 2D-systems as as compare 3D-systems, and, as the consequence, in larger friction. Sec.6 presents the conclusions and the outlook. Appendix A contains the details of the derivations of the Green functions of the electromagnetic field in two plane surface geometry. These Green functions are used in Secs. 3 and 4.

2 General formalism

There are two approaches to the theories of the van der Waals interaction and the van der Waals friction. In the first approach the fluctuating electromagnetic field is considered as a classical field which can be calculated from Maxwell's equation with the fluctuating current density as the source of the field, and with appropriate boundary conditions. This approach was used by Lifshitz in the theory of the van der Waals interaction [19] and by

Volokitin and Persson for the van der Waals friction [10, 22]. The calculation of the van der Waals friction is more complicated than of the van der Waals force because it requires the determination of the electromagnetic field between moving boundaries. The solution can be found by writing the boundary conditions on the surface of each body in the rest reference frame of this body. The relation between the electromagnetic fields in the different reference frames is determined by the Lorenz transformation. The advantage of this approach is in that, in principle, it can be used for the calculation of friction at the arbitrary relative velocities. However, the calculations become very complicated for bodies with complex geometry. At present the solutions are known for the van der Waals friction between two parallel plane surfaces [10, 22], and between a small particle and plane surface [27].

In the second approach the electromagnetic field is treated in the frame of the quantum field theory [36]. This approach was used in Ref.[20] to obtain the van der Waals interaction for an arbitrary inhomogeneous medium all parts of which are at rest.

For two bodies in slow uniform relative motion (velocity $\mathbf{v}$) the force acting on either body may be written as $\mathbf{F} = \mathbf{F}_0 - \overleftrightarrow{\mathbf{\Gamma}} \cdot \mathbf{v}$, where the adiabatic force $\mathbf{F}_0$ is independent of $\mathbf{v}$, and $\overleftrightarrow{\mathbf{\Gamma}}$, the so-called friction tensor, is defined by

$$\overleftrightarrow{\mathbf{\Gamma}} = (k_B T)^{-1} \mathrm{Re} \int_0^\infty dt \left\langle \hat{\mathbf{F}}(t) \hat{\mathbf{F}}(0) \right\rangle \tag{1}$$

Here $\langle ... \rangle$ represents a thermal average of the fluctuating force in the equilibrium state at fixed separation d between bodies, and $\hat{\mathbf{F}}(t)$ is the force operator in the Heisenberg representation. Eq. (1) is a consequence of the fluctuation-dissipation theorem [37]. For the interaction between a localized and an extended system, Eq.(1) has been derived by several authors (Schaich 1974 [38], d'Agliano *et al* 1975 [39], Nourtier 1977 [40]) and is also valid for two extended systems. In the context of the van der Waals friction Eq.(1) was used by Schaich and Harris [28], but their treatment is incomplete.

In the case of extended systems the fluctuating force operator can be expressed through the operator of the stress tensor $\hat{\sigma}_{ik}$

$$\hat{F}_i = \int dS_k \hat{\sigma}_{ik}, \tag{2}$$

where the integration is over the surface of the one of the bodies and

$$\hat{\sigma}_{ik} = \frac{1}{4\pi} \left[E_i E_k + B_i B_k - \frac{1}{2} \delta_{ik} \left(E^2 + B^2 \right) \right] \tag{3}$$

where E_i and B_i are the electric and magnetic induction field operator, respectively. The calculation of the force-force correlation function can be done using the methods of the quantum field theory [36, 41]. Such calculations are described in Sec. 3 for two plane parallel surfaces, and in Sec.4 for a small particle and plane surface, for parallel and normal relative motion. The advantage of this approach is that it only involves finding of the Green's functions of the electromagnetic field for the equilibrium system with fixed boundaries. Thus, this approach can be easily extended to bodies with complex geometry. However, it is restricted to small relative velocities.

3 Van der Waals friction between two plane surface

3.1 Parallel relative motion

Assume that the xy-plane coincides with one of the surface. For parallel relative motion the friction coefficient $\Gamma_{\parallel} = \Gamma_{xx} = \Gamma_{yy}$. Using the methods of quantum field theory [36] the expression for the friction coefficient (1) for parallel relative motion can be written in the form

$$\Gamma_{\parallel} = \lim_{\omega_0 \to 0} \mathrm{Im} \frac{G^R_{xx}(\omega_0 + i\delta)}{\omega_0}, \tag{4}$$

where G^R_{xx} is the retarded Green's function determined by

$$G^R_{xx}(\omega) = \frac{i}{\hbar} \int_0^\infty dt e^{i\omega t} \left\langle \hat{F}_x(t)\hat{F}_x(0) - \hat{F}_x(0)\hat{F}_x(t) \right\rangle \tag{5}$$

where

$$\hat{F}_x = \int dS_z \hat{\sigma}_{xz}, \tag{6}$$

where the surface integral is taken over the surface of the body at $z = 0$,

$$\hat{\sigma}_{xz} = \left(E_x E_z + E_z E_x + B_x B_z + B_z B_x \right) / 8\pi \tag{7}$$

The function G^R_{xx} can be obtained by analytic continuation in the upper half plane of the temperature Green's function G_{xx} determined on the discrete set of point $i\omega_n = i2\pi n/\beta$ by the formula

$$G_{xx}(i\omega_n) = -\frac{1}{\hbar} \int_0^\beta d\tau e^{i\omega_n \tau} \left\langle T_\tau \hat{F}_x(\tau) \hat{F}_x(0) \right\rangle, \tag{8}$$

where n is an integer and $\beta = \hbar/k_BT$. T_τ is the time- ordering operator. The function $G_{xx}(i\omega_n)$ can be calculated using standard techniques of quantum field theory [36, 41]. Thus, the function $G_{xx}(i\omega_n)$ can be represented through the Green's functions of the electromagnetic field

$$D_{ij}^{EE}(\mathbf{r},\mathbf{r}',i\omega_n) = D_{ij}(\mathbf{r},\mathbf{r}',i\omega_n) = -\frac{1}{\hbar}\int_0^\beta d\tau e^{i\omega_n\tau}\left\langle T_\tau \hat{E}_i(\tau)\hat{E}_j(0)\right\rangle, \qquad (9)$$

where the retarded Green functions $D_{ij}(\mathbf{r},\mathbf{r}',\omega)$ obey the equations [36]

$$\begin{aligned} & \left(\nabla_i\nabla_k - \delta_{ik}\nabla^2\right) D_{kj}(\mathbf{r},\mathbf{r}',\omega) - (\omega/c)^2\int d^3x''\epsilon_{ik}(\mathbf{r},\mathbf{r}'',\omega)D_{kj}(\mathbf{r}'',\mathbf{r}',\omega) \\ = \; & (4\pi\omega^2/c^2)\delta_{ij}\delta(\mathbf{r}-\mathbf{r}') \end{aligned} \qquad (10)$$

$$\begin{aligned} & \left(\nabla_j'\nabla_k' - \delta_{jk}\nabla'^2\right) D_{ik}(\mathbf{r},\mathbf{r}',\omega) - (\omega/c)^2\int d^3x''\epsilon_{kj}(\mathbf{r}'',\mathbf{r}',\omega)D_{ik}(\mathbf{r},\mathbf{r}'',\omega) \\ = \; & (4\pi\omega^2/c^2)\delta_{ij}\delta(\mathbf{r}-\mathbf{r}') \end{aligned} \qquad (11)$$

For the plane surface it is convenient to decompose the electromagnetic field into s- and $p-$ polarized plane waves. In this representation with $\hat{q} = \mathbf{q}/q$ and $\hat{n} = [\hat{z}\times\hat{q}]$, where $\mathbf{q}$ is the surface component of the wave vector, the Green's tensor is given by

$$\begin{aligned} \overleftrightarrow{\mathbf{D}}^{EE}(\mathbf{r},\mathbf{r}') = \int \frac{d^2q}{(2\pi)^2}\Big(& \hat{n}D_{nn}^{EE}(z,z',\mathbf{q})\hat{n} + \hat{q}D_{qq}^{EE}(z,z',\mathbf{q})\hat{q} + \hat{z}D_{zz}^{EE}(z,z',\mathbf{q})\hat{z} \\ & + \hat{z}D_{zq}(z,z',\mathbf{q})\hat{q} + \hat{q}D_{qz}(z,z',\mathbf{q})\hat{z}\Big)e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')} \end{aligned} \qquad (12)$$

where we have taken into account that $D_{nz}^{EE} = D_{nq}^{EE} = 0$ (see Appendix A). For two plane surface geometry the solution of Eqs.(10, 11) is described in Appendix A. Using the methods of the quantum field theory [36, 41] for the Green function G_{xx} we get

$$\begin{aligned} G_{xx}(i\omega_n) = \frac{\hbar A}{16\pi^2\beta}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\sum_{\omega_m}\frac{q_x^2}{q^2}\Big[& D_{qq}^{EE}(\mathbf{q},i\omega_m,z,z')D_{zz}^{EE}(-\mathbf{q},i\omega_n - i\omega_m,z,z') \\ & + D_{qz}^{EE}(\mathbf{q},i\omega_m,z,z')D_{zq}^{EE}(-\mathbf{q},i\omega_n - i\omega_m,z,z') \\ & + D_{qq}^{BB}(\mathbf{q},i\omega_m,z,z')D_{zz}^{BB}(-\mathbf{q},i\omega_n - i\omega_m,z,z') \\ & + D_{qz}^{BB}(\mathbf{q},i\omega_m,z,z')D_{zq}^{BB}(-\mathbf{q},i\omega_n - i\omega_m,z,z')\Big]_{z=z'=0} \end{aligned} \qquad (13)$$

where A is the surface area, and $D_{ij}^{BB}(\mathbf{q}, i\omega_m, z, z')$ is given by [36]

$$D_{ij}^{BB}(\mathbf{r}, \mathbf{r}', i\omega_n) = -\left(\frac{c}{\omega_n}\right)^2 \mathrm{rot}_{ik}\mathrm{rot}'_{jl} D_{kl}^{EE}(\mathbf{r}, \mathbf{r}', i\omega_n) \tag{14}$$

In Eq.(13) we omitted terms involving product of Green's functions associated with the p- and s- polarized electromagnetic field because after summation they cancel to each other. Even without any detailed calculations it is clear that such terms must give zero contribution to the friction because the p- and s- polarized waves must give independent contributions to the friction.

All the sums over ω_m in Eq.(13) can be calculated in a similar way. Thus, as illustration, we consider only one sum:

$$\frac{1}{\beta}\sum_{\omega_m} D_{qz}^{EE}(\mathbf{q}, \omega_m) D_{zq}^{EE}(-\mathbf{q}, i\omega_n - i\omega_m) \tag{15}$$

According to the Lehmann representation, the Green's function can be written in the form

$$D_{\alpha\beta}^{EE}(\omega_n, \mathbf{r}, \mathbf{r}') = \frac{1}{\pi}\int_{-\infty}^{\infty} dx \frac{\rho_{\alpha\beta}^{EE}(x, \mathbf{r}, \mathbf{r}')}{x - i\omega_n} \tag{16}$$

where

$$\rho_{\alpha\beta}^{EE}(\omega, \mathbf{r}, \mathbf{r}') = \sum_{n,m} \exp(F - E_n)(E_\alpha(\mathbf{r}))_{nm}(E_\alpha(\mathbf{r}'))_{mn}(1 - e^{-\beta\omega_{mn}})\delta(\omega - \omega_{mn})$$

Using (16) and standard rules for the evaluation of the sum like (15) [41] we get

$$\begin{aligned} &\frac{1}{\beta}\sum_{\omega_m} D_{qz}^{EE}(\mathbf{q}, \omega_m) D_{zq}^{EE}(-\mathbf{q}, i\omega_n - i\omega_m) \\ &= \int_{-\infty}^{\infty} d\omega \Big[\Big(\rho_{qz}^{EE}(\mathbf{q}, \omega) D_{zq}^{EE}(-\mathbf{q}, i\omega_n - \omega)\Big) n(\omega) \\ &\quad + \Big(D_{qz}^{EE}(\mathbf{q}, i\omega_n - \omega)\rho_{zq}^{EE}(-\mathbf{q}, \omega)\Big)(n(\omega) + 1)\Big] \end{aligned} \tag{17}$$

Using Eqs.(66) and (67) in (17) we get

$$\frac{\hbar}{\beta}\sum_{\omega_m} D_{qz}^{EE}(\mathbf{q}, \omega_m) D_{zq}^{EE}(-\mathbf{q}, i\omega_n - i\omega_m) =$$

$$-q^2\frac{\hbar}{\pi}\int_{-\infty}^{\infty}d\omega\Big[\left(\frac{\partial}{\partial z'}\frac{\rho_{qq}^{EE}(\omega,z,z')}{\gamma^2(\omega)}\frac{\partial}{\partial z}\frac{D_{qq}(i\omega_n-\omega,z,z')}{\gamma^2(i\omega_n-\omega)}\right)n(\omega)$$

$$+\left(\frac{\partial}{\partial z'}\frac{D_{qq}^{EE}(i\omega_n-\omega,z,z')}{\gamma^2(i\omega_n-\omega)}\frac{\partial}{\partial z}\frac{\rho_{qq}^{EE}(\omega,z,z')}{\gamma^2(\omega)}\right)(n(\omega)+1)\Big] \qquad (18)$$

where $\gamma^2(\omega)=(\omega/c)^2-q^2$. Replacing $i\omega_n\to\omega_0+i\delta$ and taking the imaginary part of Eq.(18) gives, in the limit $\omega_0\to 0$, the following contribution to the friction coming from Eq.(18)

$$\lim_{\omega_0\to 0}\mathrm{Im}\frac{1}{\omega_0}\lim_{i\omega_n\to\omega_0+i\delta}\frac{\hbar}{\beta}\sum_{\omega_m}D_{qz}^{EE}(\mathbf{q},\omega_m)D_{zq}^{EE}(-\mathbf{q},i\omega_n-i\omega_m)=$$

$$-\frac{2\hbar q^2}{\pi\gamma^4}\int_{-\infty}^{\infty}d\omega\left(-\frac{\partial n}{\partial\omega}\right)\left(\frac{\partial}{\partial z}\ \mathrm{Im}D_{qq}(\omega)\right)\left(\frac{\partial}{\partial z'}\ \mathrm{Im}D_{qq}(\omega)\right) \qquad (19)$$

Performing a similar calculation of the other sums in Eq(13) gives

$$\gamma_{xx}=\frac{\hbar}{8\pi^3}\int_0^{\infty}d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{q_x^2}{q^2}\Big\{\Big[\mathrm{Im}D_{qq}\mathrm{Im}D_{zz}$$

$$-\frac{q^2}{\gamma^4}\left(\frac{\partial}{\partial z}\mathrm{Im}D_{qq}\right)\left(\frac{\partial}{\partial z'}\mathrm{Im}D_{qq}\right)\Big]+\left(\frac{c}{\omega}\right)^4q^2\Big[\mathrm{Im}D_{nn}\frac{\partial^2}{\partial z\partial z'}\mathrm{Im}D_{nn}$$

$$-\left(\frac{\partial}{\partial z}\mathrm{Im}D_{nn}\right)\left(\frac{\partial}{\partial z'}\mathrm{Im}D_{nn}\right)\Big]\Big\}_{z=z'=0} \qquad (20)$$

Using Eqs.(66,67,71,73) for Green's functions in Eq.(20), the contribution to the friction from the propagating ($q<\omega/c$) waves becomes:

$$\gamma_{\parallel}^{rad}=\frac{\hbar}{8\pi^3}\int_0^{\infty}d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{q<\omega/c}d^2\mathbf{q}q_x^2\times$$

$$\Big[\mathrm{Re}\left(\frac{1+R_{1p}R_{2p}e^{2i\gamma d}-R_{1p}-R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)\mathrm{Re}\left(\frac{1+R_{1p}R_{2p}e^{2i\gamma d}+R_{1p}+R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)-$$

$$\left(\mathrm{Im}\frac{R_{1p}-R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)^2+[p\to s]\Big]$$

$$=\frac{\hbar}{8\pi^2}\int_0^{\infty}d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_0^{\omega/c}dqq^3\frac{(1-|R_{1p}|^2)(1-|R_{2p}|^2)}{|1-e^{2i\gamma d}R_{1p}R_{2p}|^2}+[p\to s] \qquad (21)$$

Similarly, the contribution to the friction from the evanescent electromagnetic waves ($q > \omega/c$):

$$\gamma_{\parallel}^{evan} = \frac{\hbar}{8\pi^3}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{q<\omega/c} d^2\mathbf{q}q_x^2$$

$$\times\Big[-\mathrm{Im}\left(\frac{2R_{1p}R_{2p}e^{-2|\gamma|d}-R_{1p}-R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)\mathrm{Im}\left(\frac{2R_{1p}R_{2p}e^{-2|\gamma|d}+R_{1p}+R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)$$

$$-\left(\mathrm{Im}\frac{R_{1p}-R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)^2+[p\to s]\Big]$$

$$= \frac{\hbar}{2\pi^2}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{\omega/c}^\infty dqq^3e^{-2|\gamma|d}\frac{\mathrm{Im}R_{1p}\mathrm{Im}R_{2p}}{|1-e^{-2|\gamma|}dR_{1p}R_{2p}|^2}+[p\to s] \quad (22)$$

Eqs.(21,22) were first derived in Ref.[22] using the dynamical modification of the Lifshitz theory with use of the fluctuation-dissipation theorem.

3.2 Normal relative motion

For two plane surfaces in normal relative motion the operator of the force is given by

$$\hat{F}_z = \int dS_z\hat{\sigma}_{zz}, \quad (23)$$

where

$$\hat{\sigma}_{zz} = \left(E_zE_z - E_xE_x - E_yE_y + B_zB_z - B_xB_x - B_yB_y\right)/8\pi \quad (24)$$

The friction coefficient for normal relative motion can be obtained from the analytical continuation of the Green function $G_{zz}(i\omega_m)$ which is determined by

$$G_{zz}(i\omega_n) = \frac{\hbar A}{32\pi^2\beta}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\sum_{\omega_m}\frac{q_x^2}{q^2}\Big[D_{zz}^{EE}D_{zz}^{EE}+D_{qq}^{EE}D_{qq}^{EE}+D_{nn}^{EE}D_{nn}^{EE}$$

$$-D_{zq}^{EE}D_{zq}^{EE}-D_{qz}^{EE}D_{qz}^{EE}-D_{zn}^{EB}D_{zn}^{EB}-D_{nz}^{EB}D_{nz}^{EB}$$

$$+D_{qn}^{EB}D_{qn}^{EB}+D_{nq}^{EB}D_{nq}^{EB}+[E\leftrightarrow B]\Big] \quad (25)$$

where the arguments of the Green functions in Eq.(25) are the same as in Eq.(13), $[E \leftrightarrow B]\Big]$ denotes the terms which can be obtained from the first

terms by permutation of E and B in the upper case indexes of the Green functions, and

$$D_{ij}^{EB}(\mathbf{r},\mathbf{r}',\omega_n)=\frac{c}{\omega_n}\mathrm{rot}'_{jl}D_{ij}^{EE}(\mathbf{r},\mathbf{r}',\omega_n) \tag{26}$$

$$D_{ij}^{BE}(\mathbf{r},\mathbf{r}',\omega_n)=-\frac{c}{\omega_n}\mathrm{rot}_{il}D_{lj}^{EE}(\mathbf{r},\mathbf{r}',\omega_n) \tag{27}$$

Performing similar calculations as for the parallel relative motion we get

$$\gamma_\perp=\frac{\hbar}{16\pi^3}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}$$

$$\times\Big\{\Big[\left(\mathrm{Im}D_{qq}\right)^2+\frac{\gamma^4}{q^4}\left(\mathrm{Im}D_{zz}\right)^2+\frac{2}{\gamma^2}\left(\frac{\partial}{\partial z}\mathrm{Im}D_{zz}\right)^2\Big]$$

$$+\left(\frac{c}{\omega}\right)^4\Big[\gamma^4\left(\mathrm{Im}D_{nn}\right)^2+\left(\frac{\partial^2}{\partial z\partial z'}\mathrm{Im}D_{nn}\right)^2$$

$$+2\gamma^2\left(\frac{\partial}{\partial z'}\mathrm{Im}D_{nn}(z,z')\right)^2\Big]\Big\}_{z=z'=0} \tag{28}$$

Substitution the expressions for the Green's functions from Eqs.(66,67,71,73) in Eq.(28) gives the contribution to the friction from propagating waves:

$$\gamma_\perp^{rad}=\frac{\hbar}{16\pi^3}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{q<\omega/c}d^2\mathbf{q}\gamma^2\times$$

$$\Big[\left(\mathrm{Re}\frac{1+R_{1p}R_{2p}e^{2i\gamma d}-R_{1p}-R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)^2+\left(\mathrm{Re}\frac{1+R_{1p}R_{2p}e^{2i\gamma d}+R_{1p}+R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)^2$$

$$+2\left(\mathrm{Im}\frac{R_{1p}-R_{2p}e^{2i\gamma d}}{1-e^{2i\gamma d}R_{1p}R_{2p}}\right)^2+[p\rightarrow s]\Big]=\frac{\hbar}{4\pi^2}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_0^{\omega/c}dqq\gamma^2$$

$$\times\frac{(1-|R_{1p}|^2|R_{2p}|^2)^2+|(1-|R_{1p}|^2)R_{2p}e^{i\gamma d}+(1-|R_{2p}|^2)R_{1p}^*e^{-i\gamma d}|^2}{|1-e^{2i\gamma d}R_{1p}R_{2p}|^4}+[p\rightarrow s] \tag{29}$$

In similar way one can obtain the contribution to the friction from the evanescent electromagnetic waves:

$$\gamma_\perp^{evan}=\frac{\hbar}{4\pi^3}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{q>\omega/c}d^2\mathbf{q}|\gamma|^2$$

$$\times\Big[\left(\mathrm{Im}\frac{2R_{1p}R_{2p}e^{-2|\gamma|d}-R_{1p}-R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)^2+\left(\mathrm{Im}\frac{2R_{1p}R_{2p}e^{-2|\gamma|d}+R_{1p}+R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)^2$$

$$-2\left(\mathrm{Im}\frac{R_{1p}-R_{2p}e^{-2|\gamma|d}}{1-e^{-2|\gamma|d}R_{1p}R_{2p}}\right)^2+[p\rightarrow s]\Big]=\frac{\hbar}{\pi^2}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\int_{\omega/c}^\infty dqq|\gamma|^2e^{-2|\gamma|d}$$

$$\times[(\mathrm{Im}R_{1p}+e^{-2|\gamma|d|}|R_{1p}|^2\mathrm{Im}R_{2p})(\mathrm{Im}R_{2p}+e^{-2|\gamma|d|}|R_{2p}|^2\mathrm{Im}R_{1p})$$

$$+e^{-2|\gamma|d}(\mathrm{Im}(R_{1p}R_{2p}))^2]\frac{1}{|1-e^{-2|\gamma|d}R_{1p}R_{2p}|^4}+[p\rightarrow s] \qquad (30)$$

Eqs.(29,30) were first presented without derivation in Ref.[9]. The derivation was given in Ref.[10] on the base of the dynamical modification of the semi-classical Lifshitz theory [19] of the van der Waals interaction and the Rytov theory [42, 43, 44] of the fluctuating electromagnetic field.

4 Van der Waals friction between a small particle and plane surface obtained using quantum field theory

4.1 Parallel relative motion

Let us assume that the xy- plane coincides with surface of the body and that the z- axes is directed along the upward normal. For parallel relative motion the friction coefficient $\Gamma_{\|}=\Gamma_{xx}=\Gamma_{yy}$. The Lorentz force acting on a small particle located at point $\mathbf{r}_0$ can be written in the form

$$\hat{F}_x=\left[p_k\frac{\partial}{\partial x_k}E_x(\mathbf{r})+\frac{1}{c}\left(j_yB_z-j_zB_y\right)\right]_{\mathbf{r}=\mathbf{r}_0} \qquad (31)$$

where $\mathbf{p}$ and $\mathbf{j}$ are the dipole moment and current operators of the particle, respectively. $\mathbf{E}$ and $\mathbf{B}$ are the external electric and magnetic induction field operators, respectively. The interaction of the electromagnetic field with the particle is described by the Hamiltonian

$$H_{int}=-\frac{1}{c}\mathbf{A}(\mathbf{r}_0)\cdot\mathbf{j} \qquad (32)$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential operator. Taking into account that

$$\mathbf{j}=\frac{\partial}{\partial t}\mathbf{p} \qquad (33)$$

$$\nabla \times \mathbf{E} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} \tag{34}$$

one can prove that the friction coefficient is determined by Eq.(4) where

$$G^R_{xx}(\omega) = \frac{i}{\hbar}\int_0^\infty dt e^{i\omega t}$$
$$\times \Big\langle p_k(t)\frac{\partial}{\partial x}E_k(\mathbf{r},t)p_l(0)\frac{\partial}{\partial x'}E_l(\mathbf{r}',0) - p_l(0)\frac{\partial}{\partial x'}E_l(\mathbf{r}',0)p_k(t)\frac{\partial}{\partial x}E_k(\mathbf{r},t)\Big\rangle_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \tag{35}$$

where summation over repeated indexes is assumed. After similar calculations as in Sec.3 we get

$$\Gamma_\parallel = \frac{2\hbar}{\pi}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial \omega}\right)\Big\{\sum_{k=x,y,z}\mathrm{Im}\alpha_{kk}\frac{\partial^2}{\partial x\partial x'}\mathrm{Im}D_{kk}(\mathbf{r},\mathbf{r}',\omega)$$
$$-2\mathrm{Re}\left(\alpha_{xx}(\omega)\alpha^*_{zz}(\omega)\right)\left(\frac{\partial}{\partial x}\mathrm{Im}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right)^2\Big\}_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \tag{36}$$

where $D_{ij}(\mathbf{r},\mathbf{r}')$ are the Green's functions of the electromagnetic field for one plane surface geometry without interaction with a particle. These Green functions can be obtained from the Green functions for two plane surface geometry (see Appendix A) by putting $R_{p(s)} = 0$. The polarizability of the particle

$$\alpha_{kk}(\omega) = \frac{i}{\hbar}\int_0^\infty dt e^{i\omega t} < p_k(t)p_k(0) - p_k(0)p_k(t) > \tag{37}$$

is expressed through its value α^0_{kk} in the absence of interaction between the particle and the surface

$$\alpha_{ii}(\omega) = \frac{\alpha^0_{ii}(\omega)}{1-\alpha^0_{ii}(\omega)D_{ii}(\mathbf{r}_0,\mathbf{r}_0)}. \tag{38}$$

We have also used the identity

$$\mathrm{Im}\alpha_{xx}(\omega)\mathrm{Im}\left[\alpha_{zz}(\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right]$$
$$+\mathrm{Im}\alpha_{zz}(\omega)\mathrm{Im}\left[\alpha_{xx}(\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right]$$
$$-2\mathrm{Im}\left[\alpha_{xx}(\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right]\mathrm{Im}\left[\alpha_{zz}(\omega)\frac{\partial}{\partial x}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right]$$
$$= 2\mathrm{Re}\left(\alpha_{xx}(\omega)\alpha_{zz}(\omega)^*\right)\left(\frac{\partial}{\partial x}\mathrm{Im}D_{xz}(\mathbf{r},\mathbf{r}_0,\omega)\right)^2 \tag{39}$$

4.2 Normal relative motion

The friction coefficient for a particle moving normal to the sample surface can be obtained from calculations very similar to those for parallel relative motion. In this case the Green's function G^R_{xx} must be replaced by G^R_{zz} where

$$G^R_{zz}(\omega) = \frac{i}{\hbar}\int_0^\infty dt e^{i\omega t}$$

$$\times\left\langle p_k(t)\frac{\partial}{\partial z}E_k(\mathbf{r},t)p_l(0)\frac{\partial}{\partial z'}E_l(\mathbf{r}',0) - p_l(0)\frac{\partial}{\partial z'}E_l(\mathbf{r}',0)p_k(t)\frac{\partial}{\partial z}E_k(\mathbf{r},t)\right\rangle_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \tag{40}$$

After similar calculations as in Sec.4.1 we get

$$\Gamma_\perp = \frac{2\hbar}{\pi}\int_0^\infty d\omega\left(-\frac{\partial n}{\partial\omega}\right)\sum_{k=x,y,z}\Big\{\mathrm{Im}\alpha_{kk}(\omega)\frac{\partial^2}{\partial z\partial z'}\Big[\mathrm{Im}D_{kk}(\mathbf{r},\mathbf{r}',\omega)$$

$$+\mathrm{Im}\left(\alpha_{kk}D_{kk}(\mathbf{r},\mathbf{r}_0,\omega)D_{kk}(\mathbf{r}',\mathbf{r}_0,\omega)\right)\Big]+\Big[\frac{\partial}{\partial z}\mathrm{Im}\left(\alpha_{kk}(\omega)D_{kk}(\mathbf{r},\mathbf{r}_0,\omega)\right)\Big]^2\Big\}_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0} \tag{41}$$

For a spherical particle with radius R Eq.(41) is only valid if $R \ll d$. In non-resonant case $\alpha^0_{kk} \sim R^3$ and $D_{kk} \sim d^{-3}$. Thus in this case we can neglect by the screening effects. Than taking into account that for a spherical particle $\alpha_{kk} = \alpha$, and using in the non-retarded limit (which can be formally obtained as a limit $c \to \infty$) formula (see [27] and also Appendix A)

$$\sum_{k=x,y,z} D_{kk}(\mathbf{r},\mathbf{r}',\omega) = 4\pi\int\frac{d^2qq}{(2\pi)^2}\left[e^{-q|z-z'|} + R_p(q,\omega)e^{-q(z+z')}\right]e^{i\mathbf{q}(\mathbf{x}-\mathbf{x}')} \tag{42}$$

we get

$$\Gamma_\parallel = 2\frac{\hbar}{\pi}\int_0^\infty \mathrm{d}\omega\left(-\frac{\partial n(\omega)}{\partial\omega}\right)\int_0^\infty \mathrm{d}qq^4e^{-2qd}\mathrm{Im}R_p(q,\omega)\mathrm{Im}\alpha(\omega) \tag{43}$$

and $\Gamma_\perp = 2\Gamma_\parallel$. Eqs.(36,particle9) were first obtained in Ref.[27] with use of the dynamical modification of the semiclassical Rytov theory [42, 43, 44] of the fluctuating electromagnetic field. The particular case of these equations given by Eq.(43) was first derived in Ref.[33] using the fluctuation-dissipation theorem.

5 Van der Waals friction between dielectrics and two-dimensional systems

In Refs.[34, 10] it was shown that the van der Waals friction between good conductors ($k_B T/4\pi\sigma \gg 1$, where σ is the conductivity) is extremely small. However the van der Waals friction can be greatly enhanced for high resistivity materials. Recently [35] the calculations were published, where the authors claim that they can explain the experimental data about long-range noncontact friction observed by Stipe *et.al* [6]. In this experiment the substrate did not consist of a bulk conductor but of a metal film with a thickness of a few hundred nanometers deposited on top of a dielectric substrate. Thus in the theory from Ref.[35] it was proposed that experimental measurements performed on metal films do not reflect the properties of the metal but of the underlying dielectric substrate. Using the formula (43) with the particle polarazability

$$\alpha(\omega) = R^3 \frac{\varepsilon - 1}{\varepsilon + 2} \tag{44}$$

and the reflection coefficient in the electrostatic limit ($q \gg \omega/c$)

$$R_p = \frac{\varepsilon - 1}{\varepsilon + 1}, \tag{45}$$

for high- resistivity material ($4\pi\sigma << k_B T/\hbar$) we get:

$$\Gamma_{\parallel} = \frac{9}{2} \frac{k_B T}{4\pi\sigma} \frac{R^3}{d^5} \frac{1}{2\varepsilon' + 3} \tag{46}$$

where ε' is the real part of the dielectric function $\varepsilon = \varepsilon' + 4\pi i\sigma/\omega$. In the frequency range below 0.75×10^{12}Hz the real part of the dielectric constant of glass is nearly constant ($\varepsilon' \approx 3.82$), and $\sigma = 30\text{s}^{-1}$. It was assumed that $d = s + h_s + h_t$, where s is the tip's apex-surface separation, and h_s and h_t are the thicknesses of the gold films which were deposited on the mica substrate and on the silicon cantilever, respectively. In experiment [6] $h_s = 250$nm, $h_t = 200$nm, $R = 1\mu$m, than for $s = 10$nm and $T = 300$K we get $\Gamma_{\parallel} = 7 \times 10^{-13}$kg/s, which agrees exactly with value obtained numerically in [35], and in rough agreement with the experimental value reported in Ref.[6]. However the macroscopic theory which was used in obtaining Eq.(46) is only valid when the average separation between electrons is much smaller than length scale of variation of the electric field, which is determined by

separation d. Thus the lowest value of the electron concentration n_{min} is restricted by the condition $n_{min} \geq d^{-3}$ and according to Drude formula the lowest value of the conductivity $\sigma \geq \sigma_{min} \sim e^2\tau/md^3$. For $d \sim 1\mu$m and $\tau = 10^{-15}$ the conductivity must be at least four orders of magnitude larger than it was used in [35]. For such conductivity Eq.(46) gives $\Gamma_{\|} \sim 10^{-16}$kg/s, which is three orders of the magnitude smaller than the observed friction.

Besides, in the experiment [6] the mica substrate and silica tip were coated by gold films thick enough to completely screen the electrodynamic interaction between underlying dielectrics. If planar film with thickness h and dielectric function $\varepsilon_3(\omega)$ lies on a substrate with dielectric function $\varepsilon_2(\omega)$, the system response is similar to the single-interface case if one replace the reflection coefficients R_{p21} by

$$R_p = \frac{R_{p31} - R_{p32}\exp(-2qh)}{1 - R_{p31}R_{p32}\exp(-2qh)} \tag{47}$$

where

$$R_{pij} = \frac{\varepsilon_j - \varepsilon_i}{\varepsilon_j + \varepsilon_i} \tag{48}$$

. The magnitude of the wave vector q is determined by the separation between metallic films. Thus $qh \sim h/s \gg 1$, and in this case reflection coefficient $R_p \approx R_{31}$ instead of R_{21} as it was proposed in Ref.[35].

For high-resistivity metals $(k_BT/4\pi\hbar\sigma > 1)$, for $d < c(\hbar/4\pi\sigma/ll1$ the reflection coefficient (**??**) in Eq.(22) for two surfaces in parallel relative we get

$$\gamma_{\|} \approx 0.05\frac{\hbar}{d^4}\frac{k_BT}{4\pi\hbar\sigma} \tag{49}$$

and $\gamma_{\perp} \approx 10\gamma_{\|}$. From the condition of the validity of the macroscopic theory (see above) maximum of friction can be estimated as

$$\gamma_{\|max} \sim 0.1\frac{\hbar}{d^4}\frac{k_BT}{4\pi\hbar\sigma_{min}} \sim \frac{mk_BT}{4\pi e^2\tau d}. \tag{50}$$

To estimate the friction coefficient Γ for an atomic force microscope tip with the radius of curvature $R >> d$ we can use the "proximity approximation" [46, 47]. Thus the maximal friction coefficient for a spherical tip:

$$\Gamma^s_{\|max} \sim 0.1\gamma_{max}dR \sim \frac{mk_BTR}{4\pi e^2\tau}. \tag{51}$$

For $\tau \sim 10^{-16}$s, $R \sim 1\mu$m and $T = 300$K we get $\Gamma_{max} \sim 10^{-15}$kg/s. This friction is two order of magnitude smaller than the friction observed in a recent experiment at $d = 10$nm [6]. Similarly, in the case of a cylindrical tip:

$$\Gamma^c_{max} \sim \gamma_{max}\sqrt{dR}w \sim \frac{mk_BTR^{1/2}w}{4\pi e^2\tau d^{1/2}} \tag{52}$$

where w is the width of the tip. For $w = 7\mu$m, $d = 10$nm, and with the other parameters as above, the friction is of the same orders of magnitude as it was observed in the experiment [6].

Recently a large electrostatic non-contact friction between an atomic force microscope tip and thin dielectric films was observed [13]. Similarly the van der Waals friction will be also large between dielectrics with high absorption in low-frequency range. As a particular important case we consider the van der Waals friction between thin water films adsorbed on the surfaces of the transparent dielectric substrates like silica or mica. Water has an extremely large static dielectric function of around 80. The low frequency contribution to the dielectric function, responsible for this large static value, is due to relaxation of the permanent dipoles of the water molecules. It is very nicely described by the simple Debye [48] rotation relaxation. The theoretical fit of the experimental data is given by [49]:

$$\varepsilon'(\omega) = 4.35 + \frac{C}{(1 + (\omega/\omega_0)^2)} \tag{53}$$

$$\varepsilon''(\omega) = \frac{C(\omega/\omega_0)^2}{(1 + (\omega/\omega_0)^2)} \tag{54}$$

where $C = 72.24$ and $\omega_0 = 1.3 \cdot 10^{11}\text{s}^{-1}$. The fit is very good for both the real and imaginary part of the dielectric function $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ We note that water has large absorption in the radio-frequency range at $\omega \sim \omega_0$, and shows in this region of the spectrum anomalous dispersion. In this frequency range the dielectric constants ε_2 of mica or silica are nearly uniform and $|\varepsilon_3| \gg \varepsilon_2$, where ε_3 denotes the dielectric function of water. The reflection coefficient for substrate with film is given by Eq.(47). For $qh \ll 1$ and $q^{-1} \sim d \ll |\varepsilon_3|h/\varepsilon_2$ it can be approximated by equation

$$R_p \approx 1 - \frac{2}{\varepsilon_3 qh} \tag{55}$$

Substituting (55) in eq.(22) and using "proximity" approximation for friction between cylindrical atomic force microscope tip and a sample we get

$$\Gamma_{\parallel}^{c} = \frac{\pi\hbar R^{1/2}w}{6\sqrt{2}C^2h^2d^{3/2}}\left(\frac{k_BT}{\hbar\omega_0}\right)^2 \tag{56}$$

For h = 1nm and with the same parameters as above we get $\Gamma_{\parallel}^{c} = 4.8 \cdot 10^{-12}$kg/s what is one order of magnitude larger than the friction observed in Ref.[6]. It is interesting to note that the friction coefficient (56) has the same weak distance dependence as it was observed in Ref.[6].

Another enhancement mechanism of the van der Waals friction is connected with resonant photon tunneling between adsorbate vibrational modes localized on different surfaces. In [9, 10] we have shown that resonant photon tunneling between two surfaces separated by d = 1nm, and covered by a low concentration of potassium atoms, result in a friction which is six orders of the magnitude larger than for clean surfaces. The adsorbate induced enhancement of the van der Waals friction is even larger for Cs adsorption on Cu(100). In this case, even at low coverage ($\theta \sim 0.1$), the adsorbed layer exhibits an acoustic branch for vibrations parallel to the surface [45]. Thus, $\omega_{\parallel} = 0$ and according to Ref.[12] at small frequencies the reflection coefficient is given by

$$R_p = 1 - \frac{2qa\omega_q^2}{\omega^2 - \omega_q^2 + i\omega\eta} \tag{57}$$

where $\omega_q^2 = 4\pi n_a e^{*2} a q^2/M$, e^* is the ion charge and a is the separation between an ion and the image plane. Substituting Eq.(57) in Eq. (22) for

$$\frac{a}{\eta d}\sqrt{\frac{4\pi n_a e^{*2} a}{Md^2}} \ll 1,$$

and using a "proximity" approximation, for a cylindrical tip we get

$$\Gamma_{\parallel}^{c} \approx 0.68\frac{k_BTa^2R^{0.5}w}{\eta d^{5.5}} \tag{58}$$

For Cs adsorption on Cu(100) the damping parameter η was estimated in [12] as $\eta \approx 10^{11}\text{s}^{-1}$. Using this value of η in Eq.(58) for a = 2.94Å [45], $R = 1\mu$m, $w = 7\mu$m, T = 293 K at d = 10nm we get $\Gamma_{\parallel} \approx \cdot 10^{-15}$ kg/s, which is two orders of magnitude smaller than the friction observed in [6] at the same distance. However, the van der Waals friction is characterized

by a much stronger distance dependence ($\sim 1/d^{5.5}$) than observed in the experiment ($\sim 1/d^n$, where $n = 1.3 \pm 0.2$). Thus, at small distances the van der Waals friction will be much larger than friction observed in [6], and can thus be measured experimentally.

6 Summary and conclusion

We have used a quantum field theory in Matsubara formulation to calculate the van der Waals friction between two plane surfaces, and a small particle and plane surface, for parallel and normal relative motion. The friction calculated in this approach is in agreement with the friction calculated in the framework of dynamical modification of the Lifshitz theory with use of the fluctuation-dissipation theorem. This solves the problem of the applicability of the Lifshitz theory to the dynamical situation. In quantum field theory the calculation of the friction to linear order in the sliding velocity is reduced to the finding of the equilibrium Green functions which obey to the system of the Maxwell type differential equations. Thus with application of the numerical methods of classical electrodynamics this approach can be used in the calculations of the van der Waals friction between bodies with complex geometry. We show that the van der Waals friction between high-resistivity metals, dielectrics with strong absorption in radio-frequency range, and two-dimensional systems can be measured in non-contact friction experiment using state-of-the-art equipment. The theory can be used as a guide for planning and interpretation of new experiments including more sophisticated dielectrics like in Ref.[13].

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A The Green functions for two plane surface geometry

Suppose that the half-space $z < 0$ is occupied by a solid with the reflection coefficients $R_{1p}(\mathbf{q}, \omega)$ and $R_{1s}(\mathbf{q}, \omega)$ for s- and p- electromagnetic fields, respectively. Similarly, the half-space $z > d$ is occupied with a solid with

the reflection coefficients $R_{2p}(\mathbf{q}, \omega)$ and $R_{2s}\mathbf{q}, \omega)$. The region $0 < z < d$ is assumed to be vacuum. Here $\mathbf{q}$ is the surfaces component of wave vector $\mathbf{k} = (\mathbf{q}, \gamma)$, where $\gamma = ((\omega/c)^2 - q^2)^{1/2}$. Since the system is uniform in the $\mathbf{x} = (x, y)$ directions, the Green function $D_{ij}(\mathbf{r}, \mathbf{r'})$ can be represented by the Fourier integral

$$D_{ij}(\mathbf{r}, \mathbf{r'}, omega_n) = \int \frac{\mathrm{d}^2 q}{(2\pi)^2} e^{\mathbf{iq(x-x')}} D_{ij}(z, z', \mathbf{q}, i\omega_n) \tag{59}$$

In the $xy-$plane it is convenient to choose the coordinate axes along the vectors $\hat{q} = \mathbf{q}/q$ and $\hat{n} = \hat{z} \times \hat{q}$. In this coordinate system the equations (10, 11) for the Green functions become

$$\begin{aligned}
\left(\gamma^2 + \frac{\partial^2}{\partial z^2}\right) D_{nn}(z, z') &= -\frac{4\pi\omega^2}{c^2}\delta(z - z') && (60)\\
\left(\frac{\omega^2}{c^2} - \frac{\partial^2}{\partial z^2}\right) D_{qq}(z, z') - \mathrm{i}q\frac{\partial}{\partial z} D_{zq}(z, z') &= -\frac{4\pi\omega^2}{c^2}\delta(z - z') && (61)\\
\gamma^2 D_{zq}(z, z') - \mathrm{i}q\frac{\partial}{\partial z} D_{qq}(z, z') &= 0 && (62)\\
\gamma^2 D_{zz}(z, z') - \mathrm{i}q\frac{\partial}{\partial z} D_{qz}(z, z') &= -\frac{4\pi\omega^2}{c^2}\delta(z - z') && (63)\\
\gamma^2 D_{qz}(z, z') + \mathrm{i}q\frac{\partial}{\partial z'} D_{qq}(z, z') &= 0 && (64)
\end{aligned}$$

The components D_{qn}, D_{zn} of the Green function vanish, since the equations for them turn out to be homogeneous. Solving the system of Eqs.(60)-(64) amounts to solving two equations: equation (60) for D_{nn} and the equation for D_{qq} which follows from equations (61, 62)

$$\left(\gamma^2 + \frac{\partial^2}{\partial z^2}\right) D_{qq}(z, z') = -4\pi\gamma^2\delta(z - z'), \tag{65}$$

after which D_{qz}, D_{zq} and D_{zz} for $z \neq z'$ are obtained as

$$\begin{aligned}
D^R_{qz} &= -\frac{iq}{\gamma^2}\frac{\partial}{\partial z'} D_{qq}; D_{zq} = \frac{iq}{\gamma^2}\frac{\partial}{\partial z} D_{qq}; && (66)\\
D_{zz} &= \frac{q^2}{\gamma^4}\frac{\partial^2}{\partial z \partial z'} D_{qq} && (67)
\end{aligned}$$

In the vacuum gap $0 < z < d$ the solution of equation (60) has the form

$$D_{nn}(z, z') = -\frac{2\pi i \omega^2}{\gamma c^2} e^{i\gamma|z-z'|} + v_n e^{i\gamma z} + w_n e^{-i\gamma z} \tag{68}$$

At the boundaries $z = 0$ and $z = d$ the amplitude of the scattered wave is equal to amplitude of incident wave times to corresponding reflection coefficient. The Green function D_{nn} is associated with s- polarized electromagnetic field and the boundary conditions for it give

$$v_n = R_{1s}\left(w_n + \frac{2\pi i \omega^2}{\gamma c^2} e^{i\gamma z'}\right) \qquad \text{for} \qquad z = 0 \tag{69}$$

$$w_n = R_{2s} e^{2i\gamma d}\left(v_n + \frac{2\pi i \omega^2}{\gamma c^2} e^{-i\gamma z'}\right) \qquad \text{for} \qquad z = d \tag{70}$$

Using Eqs.(68–70) we get

$$\begin{aligned} D_{nn}(z, z') &= -\frac{2\pi i \omega^2}{\gamma c^2}\left\{ e^{i\gamma|z-z'|} + \frac{R_{1s}R_{2s}e^{2i\gamma d}\left(e^{i\gamma(z-z')} + e^{-i\gamma(z-z')}\right)}{\Delta_s} \right. \\ &\quad \left. + \frac{R_{1s}e^{i\gamma(z+z')} + R_{2s}e^{2i\gamma d}e^{-i\gamma(z+z')}}{\Delta_s} \right\} \qquad (71) \\ \Delta_s &= 1 - e^{2i\gamma d} R_{2s} R_{1s} \qquad (72) \end{aligned}$$

Equation (65) for D_{qq} is similar to equation (60) for D_{nn}, and the expression for D_{qq}^R can be obtained from expression (71) just by replacements of reflection coefficient

$$D_{qq} = \left(\frac{\gamma c}{\omega}\right)^2 D_{nn}\left[R_s \rightarrow -R_p\right] \tag{73}$$

In our approach the calculation of reflection coefficient for s- and p- polarized waves constitutes separate problem, which can be solved taking into account non-local effects. For the local optic case the reflection coefficients are determined by the well known Fresnel formulas

$$R_{ip} = \frac{\varepsilon_i \gamma - \gamma_i}{\varepsilon_i \gamma + \gamma_i}, \qquad R_{is} = \frac{\gamma - \gamma_i}{\gamma + \gamma_i}, \tag{74}$$

where ε_i is the complex dielectric constant for body i:

$$\gamma_i = \sqrt{\frac{\omega^2}{c^2}\varepsilon_i - q^2} \tag{75}$$

FIGURE CAPTION

Fig. 1. The electromagnetic waves emitted in the opposite direction by the body at the bottom will experience opposite Doppler shift in the reference frame in which the body at the top is at rest. Due to the frequency dispersion of the reflection coefficient these electromagnetic waves will reflect differently from the surface of the body at the top, which give rises to momentum transfer between the bodies. This momentum transfer is the origin of the van der Waals friction.

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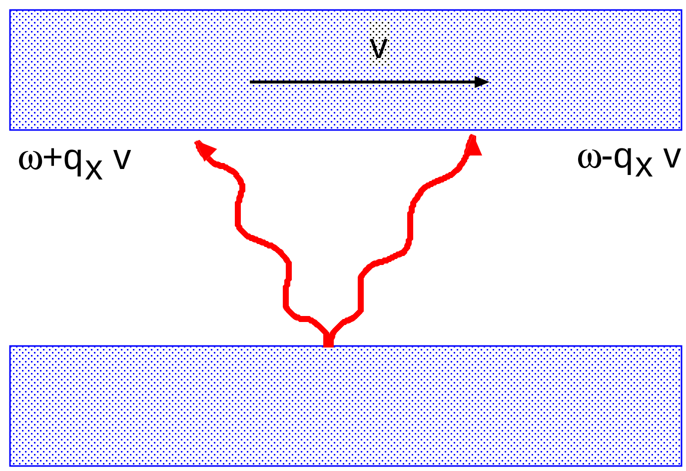
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